

# A second look at $\mathcal{N}=1$ supersymmetric $\text{AdS}_4$ vacua of type IIA supergravity

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**ABSTRACT:** We show that a class of type IIA vacua recently found within the  $D=4$  effective approach corresponds to compactification on  $\text{AdS}_4 \times \text{S}^3 \times \text{S}^3 / \mathbb{Z}_2^3$ . The results obtained using the effective method completely match the general ten-dimensional analysis for the existence of  $\mathcal{N}=1$  warped compactifications on  $\text{AdS}_4 \times \text{M}_6$ . In particular, we verify that the internal metric is nearly-Kähler and that for specific values of the parameters the Bianchi identity of the RR 2-form is fulfilled without sources. For another range of parameters, including the massless case, the Bianchi identity is satisfied when D6-branes are introduced. Solving the tadpole cancellation conditions in  $D=4$  we are able to find examples of appropriate sets of branes. In the second part of this paper we describe how an example with internal space  $\mathbb{CP}^3$  but with non nearly-Kähler metric fits into the general analysis of flux vacua.

**KEYWORDS:** D-branes, Superstring Vacua, Flux compactifications.

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## Contents

<b>1.</b>	<b>Introduction</b>	<b>1</b>
<b>2.</b>	<b>Review of supersymmetric conditions in <math>D=10</math></b>	<b>2</b>
<b>3.</b>	<b>Flux compactification on <math>\text{AdS}_4 \times \text{S}^3 \times \text{S}^3</math></b>	<b>4</b>
3.1	D6-branes on $\text{S}^3 \times \text{S}^3$ and Bianchi identity for $F_2$	8
<b>4.</b>	<b>Flux compactification on <math>\text{AdS}_4 \times \text{CP}^3</math></b>	<b>14</b>
<b>5.</b>	<b>Final remarks</b>	<b>17</b>
<b>A.</b>	<b>Effective approach in <math>D=4</math></b>	<b>18</b>
A.1	Tadpole cancellation and D6-branes	21
<b>B.</b>	<b>Supersymmetric vacua of massless type IIA supergravity in <math>D=10</math></b>	<b>24</b>

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## 1. Introduction

Four-dimensional  $\mathcal{N}=1$  supersymmetric vacua of type II supergravity with fluxes can be analyzed directly in  $D=10$  or by means of an effective potential formalism in  $D=4$ . In this work we point out that a class of type IIA vacua, with geometric fluxes switched on, that were found using the latter method [1] corresponds to compactification on  $\text{AdS}_4 \times \text{S}^3 \times \text{S}^3/\mathbb{Z}_2^3$ . The results obtained using the effective formalism are in complete accord with the general conditions for the existence of  $\text{AdS}_4 \times \text{M}_6$  vacua [2–4]. This is a particular example of the equivalence between the higher and lower dimensional approaches considered lately in greater generality [5, 6].

In the  $\text{AdS}_4 \times \text{S}^3 \times \text{S}^3/\mathbb{Z}_2^3$  compactification, that we study in depth, we show that the internal metric is nearly-Kähler. In [7] it was first proven that when  $\text{M}_6$  is nearly-Kähler there are consistent vacua of massive IIA supergravity with  $\mathcal{N}=1$  supersymmetry in  $\text{AdS}_4$ . As also remarked in [7], besides  $\text{S}^3 \times \text{S}^3$ , there are other six-dimensional compact spaces that admit a nearly-Kähler metric, namely  $\text{S}^6$ ,  $\text{CP}^3$  and  $\text{SU}(3)/\text{U}(1)^2$  [8]. However, these spaces are not group manifolds and cannot be treated in a simple effective approach based on adding geometric fluxes to a toroidal compactification. It would be interesting to formulate all nearly-Kähler compactifications within the effective four-dimensional approach. A first step in this direction is the Kaluza-Klein reduction on nearly-Kähler spaces [9]. The case of  $\text{SU}(3)/\text{U}(1)^2$  has been considered in [10].

A property of nearly-Kähler compactifications is that for special values of the fluxes the Bianchi identity for the RR 2-form can be satisfied without adding sources [7, 2]. For other

ranges of parameters it is necessary to add O6-planes, D6-branes, or both, wrapping 3-cycles in the internal space. In any case, including D6-branes is required to generate charged chiral multiplets. In the  $S^3 \times S^3/\mathbb{Z}_2^3$  compactification we will present examples of supersymmetric D6-branes that can be included to fulfill the Bianchi identity or equivalently to cancel tadpoles. This problem was first addressed in [11] where it was argued that a certain setup of D6-branes could cancel the tadpoles. We find similar results at the time we go further in proving tadpole cancellation because we supply the explicit background fluxes.

The second part of this paper is devoted to describing how other  $\mathcal{N}=1$ ,  $AdS_4$  vacua of massless IIA supergravity, discovered long time ago [12–15], fit into the modern analysis of flux vacua. In these compactifications the internal space can be  $\mathbb{C}P^3$  or  $SU(3)/U(1)^2$ , but the metric is not nearly-Kähler. We will focus on the  $\mathbb{C}P^3$  example, but the analysis can be easily extended to  $SU(3)/U(1)^2$ . We give explicit expressions for the metric and the fluxes and then find the Killing spinor that allows to derive the fundamental forms that define the  $SU(3)$  structure.

The organization of this paper is as follows. In section 2 we summarize the conditions for the existence of  $\mathcal{N}=1$   $AdS_4$  vacua derived from the  $D=10$  theory. We also discuss the issue of solving the Bianchi identity for the RR 2-form with or without sources. In section 3 we study compactification on  $AdS_4 \times S^3 \times S^3/\mathbb{Z}_2^3$  by describing the internal space in terms of a set of structure constants, the so-called geometric fluxes, known to give  $\mathcal{N}=1$  vacua from the analysis of the  $D=4$  effective potential. We then explain how the Bianchi identity for  $F_2$  can be satisfied in general by adding sources and present as well a concrete configuration of D6-branes in the massless case. There is an important interplay with the results in the  $D=4$  effective formalism that are collected in appendix A. Section 4 deals with the compactification on  $AdS_4 \times \mathbb{C}P^3$  that provides an example where the internal space is not nearly-Kähler. In appendix B we show that the proposed metric and background fluxes in  $\mathbb{C}P^3$  do satisfy the equations of motion and preserve  $\mathcal{N}=1$  supersymmetry in  $D=4$ .

## 2. Review of supersymmetric conditions in $D=10$

We are interested in  $\mathcal{N}=1$  compactifications of type IIA supergravity with fluxes turned on and warped product geometry

$$ds^2 = e^{2A(y)} ds_4^2 + ds_6^2, \tag{2.1}$$

where  $ds_4^2$  and  $ds_6^2$  are respectively the line elements of  $AdS_4$  and the internal compact space. The general conditions that these vacua must fulfill were derived in [2] using Romans massive action [16] and also in [3, 4] starting with the democratic formulation of IIA supergravity [17]. In this note we use the results and notation of [4] that are more suited to compare with the effective potential approach.

By assumption, the internal manifold has strictly  $SU(3)$  structure, i.e. it admits only one nowhere vanishing invariant spinor which in turn allows to write a fundamental 2-form  $J$  and a holomorphic 3-form  $\Omega$  satisfying the relations

$$\Omega \wedge J = 0 \quad ; \quad \Omega \wedge \Omega^* = -\frac{4i}{3} J \wedge J \wedge J. \tag{2.2}$$

In the most general supersymmetric solution of the equations of motion, the warp factor and the dilaton are constants related by  $\phi = 3A$ . Moreover, the characteristic forms  $J$  and  $\Omega$  must meet the conditions

$$dJ = 2\tilde{m}e^{-A}\text{Re}\widehat{\Omega} \quad ; \quad d\widehat{\Omega} = -\frac{4i}{3}\tilde{m}e^{-A}J^2 - i\mathcal{W}_2 \wedge J, \quad (2.3)$$

where  $\mathcal{W}_2$  is a real primitive 2-form. Here  $\widehat{\Omega} = -ie^{i(\alpha+\beta)}\Omega$ , with  $\alpha, \beta$ , phases that enter in the normalization of the  $D=10$  supersymmetry parameters (see [4] for more details). The equations of motion also require  $(\alpha - \beta)$  to be a constant.

Besides the constant  $\tilde{m}$ , the solutions depend on the IIA mass parameter  $m$ . These two real quantities are combined into the complex constant

$$\mu = e^{-i(\alpha-\beta)}(m + i\tilde{m}). \quad (2.4)$$

The parameter  $\mu$  enters in the covariant derivative of the  $D=4$  gravitino and it turns out to be related to the cosmological constant through  $\Lambda = -3|\mu|^2$ . This  $\Lambda$  is defined with respect to the unwarped  $\text{AdS}_4$  metric.

In the solution the field strengths are determined to be<sup>1</sup>

$$\begin{aligned} H &= 2me^{-A}\text{Re}\widehat{\Omega} \quad ; \quad F_0 = -5me^{-4A} \quad ; \quad F_2 = -e^{-3A} * d\text{Im}\widehat{\Omega} - 3\tilde{m}e^{-4A}J \\ F_4 &= -\frac{3}{2}me^{-4A}J^2 \quad ; \quad F_6 = \frac{1}{2}\tilde{m}e^{-4A}J^3. \end{aligned} \quad (2.5)$$

The relation to the NSNS and RR forms is given by

$$H = dB + \overline{H} \quad ; \quad F_p = dC_{p-1} - H \wedge C_{p-3} + (\overline{F} \wedge e^B)|_p. \quad (2.6)$$

The barred quantities are background fluxes and  $\overline{F} = \overline{F}_0 + \overline{F}_2 + \overline{F}_4 + \overline{F}_6$  is a formal sum.

Clearly, (2.3) implies  $J \wedge dJ = 0$  and  $d(\text{Re}\widehat{\Omega}) = 0$ . This means that the internal space is always a half-flat manifold. If the torsion class  $\mathcal{W}_2$  vanishes the internal space is nearly-Kähler and the RR 2-form simplifies to

$$F_2 = -\frac{\tilde{m}}{3}e^{-4A}J. \quad (2.7)$$

This implies in particular that  $dF_2 \neq 0$  in nearly-Kähler compactifications.

The Bianchi identities for  $H$  and  $F_4$  are automatically satisfied. On the other hand, for the RR 2-form the generic results imply  $dF_2 - F_0H \neq 0$ . The situation is not hopeless because there might be further contributions due to D6-branes or O6-planes wrapping 3-cycles in the internal space. Actually, the Bianchi identity (BI) for  $F_2$  is equivalent to tadpole cancellation conditions for the RR  $C_7$  form that couples to such sources.

Following the prescription of [4] we assume that the sources are smeared instead of localized. This means that in the BI D6-branes and O6-planes can be represented by additional 3-forms in the internal space. This is actually the only consistent possibility

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<sup>1</sup>The sign differences with respect to equation (7.9) in [4] are due to our conventions  $*J = J^2/2$  and  $*1 = J^3/6$ , where  $*$  is the Hodge dual in six dimensions.

for the AdS<sub>4</sub> vacua in which the warp factor must be constant. Upon including smeared sources the BI becomes

$$dF_2 - F_0 H + A_3 = 0, \tag{2.8}$$

where  $A_3$  is the Poincaré dual to internal 3-cycles wrapped by D6-branes or O6-planes. By virtue of (2.6), this identity can be written purely in terms of background fluxes as  $d\overline{F}_2 - F_0\overline{H} + A_3 = 0$ .

A property of the  $\mathcal{N}=1$  AdS<sub>4</sub> vacua is that  $H \propto dJ$ . Thus, the form  $A_3$  is necessarily exact. In consequence, to saturate the Bianchi identity of the RR 2-form, or equivalently to cancel  $C_7$  tadpoles, the sources need not wrap non-trivial 3-cycles. This point has been known for some time [11, 18] and further elaborated recently [19]. Due to the special properties of AdS<sub>4</sub> such D6-branes can still be stable.

When  $F_0 \neq 0$  there could be a solution of (2.8) without sources even if  $dF_2 \neq 0$ . Indeed, when the internal space is nearly-Kähler from the above results it follows that

$$dF_2 - F_0 H = \frac{2}{3} e^{-5A} (15m^2 - \tilde{m}^2) \text{Re } \widehat{\Omega}. \tag{2.9}$$

Therefore, it is possible to avoid sources, i.e.  $A_3 = 0$ , provided that  $\tilde{m}^2 = 15m^2$ . This interesting fact was first obtained in [7] and later in [2]. On the other hand, if  $\tilde{m}^2 \neq 15m^2$ , sources must be added to fulfill the Bianchi identity. For instance, if  $\tilde{m}^2 > 15m^2$  a solution can be achieved by adding only D6-branes. This follows because supersymmetric 3-cycles are calibrated by  $\text{Re } \Omega$  and in this case  $\int_{M_6} \text{Re } \Omega \wedge A_3 > 0$ . Here we are taking  $\widehat{\Omega} = -i\Omega$  according to results in appendix A.

It is also feasible to satisfy the Bianchi identity without sources and  $F_0 = 0$  simply when  $dF_2 = 0$ . Clearly, in this situation the internal space cannot be nearly-Kähler. Instead, the torsion class  $\mathcal{W}_2$  must be non-zero. Examples of this type were actually found several years ago [12–15]. In section 4 we discuss in detail the case of compactification on  $\mathbb{C}\mathbb{P}^3$ .

### 3. Flux compactification on AdS<sub>4</sub> × S<sup>3</sup> × S<sup>3</sup>

We are interested in  $\mathcal{N}=1$  type IIA vacua in presence of geometric fluxes  $\omega_{MN}^P$  together with NSNS and RR fluxes. Such solutions can be viewed as compactifications in which the internal space has a basis of globally defined 1-forms satisfying

$$d\eta^P = -\frac{1}{2} \omega_{MN}^P \eta^M \wedge \eta^N, \tag{3.1}$$

where the  $\omega_{MN}^P$  are the structure constants of some Lie group  $G$ . If the Killing form  $K_{MN} = \omega_{MR}^P \omega_{NP}^R$  is non-degenerate,  $G$  is semisimple and furthermore it is compact if  $K_{MN}$  is negative definite. If  $G$  is not semisimple, but it has a discrete compact sub-group  $\Gamma$ , the internal space can be compactified by taking the quotient  $G/\Gamma$ . This is the case of the nil and solvmanifolds studied in [4]. In this note we rather study the situation where  $G$  is compact and the internal space is the  $G$  group manifold. In particular, we want to show that in a class of supersymmetric AdS<sub>4</sub> × M<sub>6</sub> vacua found in [1] the structure constants are actually those of  $SU(2) \times SU(2)$  and the internal space is  $S^3 \times S^3$  realized as  $SU(2) \times SU(2) \times SU(2)/SU(2)_{\text{diag}}$ .

The number of independent geometric fluxes  $\omega_{MN}^P$  can be reduced by imposing additional conditions on the internal space. We will enforce a  $\mathbb{Z}_2 \times \mathbb{Z}_2$  symmetry whose generators act as

$$\begin{aligned} \mathbb{Z}_2 : (\eta^1, \eta^2, \eta^3, \eta^4, \eta^5, \eta^6) &\rightarrow (-\eta^1, -\eta^2, \eta^3, -\eta^4, -\eta^5, \eta^6) \\ \mathbb{Z}_2 : (\eta^1, \eta^2, \eta^3, \eta^4, \eta^5, \eta^6) &\rightarrow (\eta^1, -\eta^2, -\eta^3, \eta^4, -\eta^5, -\eta^6) . \end{aligned} \quad (3.2)$$

Furthermore, keeping in mind the eventual need for orientifold planes to cancel tadpoles, the geometric fluxes are required to be invariant under an orientifold involution  $\sigma$  which is also a  $\mathbb{Z}_2$  symmetry given by

$$\sigma : \eta^i \rightarrow \eta^i \quad ; \quad \eta^{i+3} \rightarrow -\eta^{i+3} \quad , \quad i = 1, 2, 3 . \quad (3.3)$$

In the end only twelve geometric fluxes survive and they are further constrained by the Bianchi identities following from (3.1). In the AdS<sub>4</sub> solutions found in [1] there are only four independent parameters  $a$  and  $b_i$  which appear in the structure equations

$$\begin{aligned} d\eta^1 &= -a\eta^{56} - b_1\eta^{23} \quad ; \quad d\eta^4 = -b_2\eta^{53} - b_3\eta^{26} \\ d\eta^2 &= -a\eta^{64} - b_2\eta^{31} \quad ; \quad d\eta^5 = -b_1\eta^{34} - b_3\eta^{61} \\ d\eta^3 &= -a\eta^{45} - b_3\eta^{12} \quad ; \quad d\eta^6 = -b_1\eta^{42} - b_2\eta^{15} . \end{aligned} \quad (3.4)$$

The notation  $\eta^{12} = \eta^1 \wedge \eta^2$ , etc. is understood.

For future purposes we record the 2, 3 and 4-forms invariant under the  $\mathbb{Z}_2 \times \mathbb{Z}_2$  symmetry. These are

$$\begin{aligned} \omega_1 = -\eta^{14} \quad ; \quad \alpha_0 = \eta^{123} \quad ; \quad \beta_0 = \eta^{456} \quad ; \quad \tilde{\omega}_1 = \eta^{2536} \\ \omega_2 = -\eta^{25} \quad ; \quad \alpha_1 = \eta^{156} \quad ; \quad \beta_1 = \eta^{423} \quad ; \quad \tilde{\omega}_2 = \eta^{1436} \\ \omega_3 = -\eta^{36} \quad ; \quad \alpha_2 = \eta^{426} \quad ; \quad \beta_2 = \eta^{153} \quad ; \quad \tilde{\omega}_3 = \eta^{1425} \\ \alpha_3 = \eta^{453} \quad ; \quad \beta_3 = \eta^{126} \quad . \end{aligned} \quad (3.5)$$

Notice that  $\alpha_I$  and  $\tilde{\omega}_i$  are even whereas  $\beta_I$  and  $\omega_i$  are odd under the orientifold involution. The normalization is

$$\int_{M_6} \alpha_i \wedge \beta_j = \int_{M_6} \omega_i \wedge \tilde{\omega}_j = \mathcal{V}_6 \delta_{ij} , \quad (3.6)$$

where  $\mathcal{V}_6$  is a constant to be computed later on.

When the geometric fluxes  $a$  and  $b_i$  are zero, the internal space can be compactified into a flat six-dimensional torus. Moreover, the  $\mathbb{Z}_2 \times \mathbb{Z}_2$  symmetry that is assumed implies that this torus is a product of three  $T_i^2$ . Each 2-torus has a basis of 1-forms  $(\eta^i, \eta^{i+1})$ , a Kähler modulus (area)  $t_i$  and a complex structure parameter  $\tau_i$  that must be real for consistency with the orientifold involution. With this picture in mind we take the metric on  $M_6$ , with  $a, b_i \neq 0$ , to still be given by

$$ds_6^2 = \sum_{i=1}^3 \frac{t_i}{\tau_i} (\eta^i)^2 + t_i \tau_i (\eta^{i+3})^2 . \quad (3.7)$$

By construction,  $t_i > 0$  and  $\tau_i > 0$ . Clearly,  $\sqrt{g_6} = t_1 t_2 t_3$ . Integrating gives the volume  $\text{Vol}(M_6) = \mathcal{V}_6 t_1 t_2 t_3$ , where  $\mathcal{V}_6$  is the normalization constant defined above.

The hermitian almost complex structure corresponding to the metric is

$$J = -t_1\eta^{14} - t_2\eta^{25} - t_3\eta^{36} = t_i\omega_i . \quad (3.8)$$

The associated holomorphic (3,0) form can be written as

$$\Omega = \sqrt{\frac{t_1 t_2 t_3}{\tau_1 \tau_2 \tau_3}} (\eta^1 - i\tau_1 \eta^4) \wedge (\eta^2 - i\tau_2 \eta^5) \wedge (\eta^3 - i\tau_3 \eta^6) . \quad (3.9)$$

These  $J$  and  $\Omega$  satisfy (2.2) so that they provide an  $SU(3)$  structure on the internal space  $M_6$ . Notice also that under the orientifold involution,  $J \rightarrow -J$  and  $\Omega \rightarrow \Omega^*$ .

From (3.4) we find that  $dJ$  and  $d\Omega$  are not zero but  $J \wedge dJ$  and  $d(\text{Im } \Omega)$  do vanish. Thus, the  $M_6$  defined by (3.4) is a half-flat manifold. Additional properties must be fulfilled for  $M_6$  to serve as internal space in an  $\mathcal{N}=1$  supersymmetric  $AdS_4$  vacua of type IIA. Moreover, it is necessary to turn on particular NSNS and RR background fluxes. Now, from the discussion in [1] we know that a solution is obtained with a precise set of fluxes invariant under the  $\Gamma = \mathbb{Z}_2^3$  group of symmetries (3.2) and (3.3). Furthermore, in this solution the variables  $t_i$  and  $\tau_i$  that enter in the metric satisfy specific relations. In the following our strategy is to use these results to continue analyzing the properties of the  $M_6$  at hand.

In the appendix we review the conditions of [1] to obtain  $AdS_4 \times M_6$  supersymmetric minima. The fluxes allow a configuration with  $t_1 = t_2 = t_3 = t$ , where  $t$  is completely fixed. A crucial property is that the structure constants  $a$  and  $b_i$  must all have the same sign. Also, the second equation in (A.9) together with the explicit form of the moduli, c.f. (A.4), gives the very useful relations

$$b_i \tau_j \tau_k = 3a \quad \Rightarrow \quad \tau_i^2 = \frac{3ab_i}{b_j b_k}, \quad i \neq j \neq k . \quad (3.10)$$

We then find

$$dJ = \frac{3}{2} \text{Im}(\mathcal{W}_1 \Omega) \quad ; \quad d\Omega = \mathcal{W}_1 J \wedge J \quad ; \quad \mathcal{W}_1 = \frac{2a}{\sqrt{t\tau_1\tau_2\tau_3}} \quad (3.11)$$

In general the exterior derivatives of  $J$  and  $\Omega$  can be expressed in terms of torsion classes (see e.g. [20]). In our case, from (3.11) we easily see that the only non-zero class is  $\mathcal{W}_1$ . This is precisely the condition for the internal space to be nearly-Kähler.

It is a simple exercise to compute the Killing form for the structure constants given in (3.4). We find

$$K = -4 \text{diag}(b_2 b_3, b_1 b_3, b_1 b_2, ab_1, ab_2, ab_3) . \quad (3.12)$$

Now, recall that to obtain  $AdS_4 \times M_6$  supersymmetric minima the geometric fluxes  $a$  and  $b_i$  must all have the same sign. Therefore,  $K$  is non-degenerate and negative-definite. We might guess that the semisimple compact algebra being six-dimensional is that of  $SU(2) \times SU(2)$ . Indeed, after performing the change of basis

$$\begin{aligned} \xi^1 &= \sqrt{b_2 b_3} \eta^1 + \sqrt{ab_1} \eta^4 & ; & \quad \hat{\xi}^1 = \sqrt{b_2 b_3} \eta^1 - \sqrt{ab_1} \eta^4 \\ \xi^2 &= \sqrt{b_1 b_3} \eta^2 + \sqrt{ab_2} \eta^5 & ; & \quad \hat{\xi}^2 = \sqrt{b_1 b_3} \eta^2 - \sqrt{ab_2} \eta^5 \\ \xi^3 &= \sqrt{b_1 b_2} \eta^3 + \sqrt{ab_3} \eta^6 & ; & \quad \hat{\xi}^3 = \sqrt{b_1 b_2} \eta^3 - \sqrt{ab_3} \eta^6 , \end{aligned} \quad (3.13)$$

the structure equations become

$$d\xi^i = -\frac{1}{2}\epsilon_{ijk}\xi^j \wedge \xi^k \quad ; \quad d\hat{\xi}^i = -\frac{1}{2}\epsilon_{ijk}\hat{\xi}^j \wedge \hat{\xi}^k . \quad (3.14)$$

This confirms that the underlying algebra is that of  $SU(2) \times SU(2)$ .

We can take the  $\xi^i$  and  $\hat{\xi}^i$  to be two sets of  $SU(2)$  left invariant 1-forms. Concretely,

$$\begin{aligned} \hat{\xi}^1 &= \cos \hat{\psi} d\hat{\theta} + \sin \hat{\psi} \sin \hat{\theta} d\hat{\phi} \\ \hat{\xi}^2 &= -\sin \hat{\psi} d\hat{\theta} + \cos \hat{\psi} \sin \hat{\theta} d\hat{\phi} \\ \hat{\xi}^3 &= d\hat{\psi} + \cos \hat{\theta} d\hat{\phi} , \end{aligned} \quad (3.15)$$

and similarly for the  $\xi^i$ . The range of angles is  $0 \leq \hat{\theta} \leq \pi$ ,  $0 \leq \hat{\phi} \leq 2\pi$  and  $0 \leq \hat{\psi} \leq 4\pi$ .

Our claim that the internal space is  $S^3 \times S^3$  is supported by the explicit form of the metric in the new basis. Substituting (3.13) into (3.7) readily gives

$$ds_6^2 = \frac{t}{\sqrt{3ab_1b_2b_3}} [(\xi^i)^2 + (\hat{\xi}^i)^2 - \xi^i \hat{\xi}^i] . \quad (3.16)$$

This is an Einstein metric that belongs to a family of homogeneous metrics on  $S^3 \times S^3$  [21]. The isometry group is  $SU(2)^3$  [22, 23]. There are two  $SU(2)$ 's from the left actions that leave  $\xi^i$  and  $\hat{\xi}^i$  separately invariant, and a further  $SU(2)$  from a simultaneous right action by the same element on  $\xi^i$  and  $\hat{\xi}^i$ . From the metric and the explicit realization of the  $SU(2)$  1-forms the volume of  $S^3 \times S^3$  can be evaluated to be

$$\text{Vol}(S^3 \times S^3) = \frac{(4\pi)^4 t^3}{(4ab_1b_2b_3)^{3/2}} \equiv \mathcal{V}_6 t^3 , \quad (3.17)$$

where  $\mathcal{V}_6$  is precisely the normalization constant introduced in (3.6).

In the new basis the fundamental forms  $J$  and  $\Omega$  are given by

$$\begin{aligned} J &= \frac{t}{2\sqrt{ab_1b_2b_3}} (\xi^1 \wedge \hat{\xi}^1 + \xi^2 \wedge \hat{\xi}^2 + \xi^3 \wedge \hat{\xi}^3) \\ \Omega &= -\frac{t^{3/2}}{(3ab_1b_2b_3)^{3/4}} (\xi^1 + e^{2i\pi/3}\hat{\xi}^1) \wedge (\xi^2 + e^{2i\pi/3}\hat{\xi}^2) \wedge (\xi^3 + e^{2i\pi/3}\hat{\xi}^3) . \end{aligned} \quad (3.18)$$

Similar expressions have appeared in the literature some time ago [11] and more recently [19].

At this point we must remember that our actual model is constrained by some specific symmetries. Indeed, the geometric fluxes (3.4), as well as the NSNS and RR backgrounds (A.5), have been chosen to be invariant under the group  $\Gamma = \mathbb{Z}_2^3$  of transformations given by the geometric  $\mathbb{Z}_2 \times \mathbb{Z}_2$  (3.2) and the orientifold involution  $\sigma$  (3.3). The action of  $\sigma$  amounts to exchange of the spheres,  $\xi^i \leftrightarrow \hat{\xi}^i$ , which is clearly a symmetry of the metric. On the other hand, the geometric  $\mathbb{Z}_2 \times \mathbb{Z}_2$  corresponds to

$$\begin{aligned} \mathbb{Z}_2 : (\xi^1, \xi^2, \xi^3, \hat{\xi}^1, \hat{\xi}^2, \hat{\xi}^3) &\rightarrow (-\xi^1, -\xi^2, \xi^3, -\hat{\xi}^1, -\hat{\xi}^2, \hat{\xi}^3) \\ \mathbb{Z}_2 : (\xi^1, \xi^2, \xi^3, \hat{\xi}^1, \hat{\xi}^2, \hat{\xi}^3) &\rightarrow (\xi^1, -\xi^2, -\xi^3, \hat{\xi}^1, -\hat{\xi}^2, -\hat{\xi}^3) \end{aligned} \quad (3.19)$$



which also leaves the metric invariant. The effect of these latter symmetries is to restrict the range of the angles that define the 1-forms, c.f. (3.15). The first and second  $\mathbb{Z}_2$ 's imply respectively  $\hat{\theta} \equiv -\hat{\theta}$ , and  $\hat{\psi} \equiv -\hat{\psi}$  simultaneously with  $\hat{\phi} \equiv -\hat{\phi}$ , and analogous for the unhatted angles. In the end we truly have internal space  $S^3 \times S^3/\Gamma$ , with volume given by  $\mathcal{V}_6 t^3/8$ . We will write

$$\text{Vol}(S^3 \times S^3/\Gamma) = \mathcal{C} t^3, \tag{3.20}$$

where  $\mathcal{C} = \mathcal{V}_6/8 = 4\pi^4/(ab_1b_2b_3)^{3/2}$ .

The nearly-Kähler metric on  $S^3 \times S^3$  is also invariant under the order three transformation

$$\beta : \xi^i \rightarrow -\hat{\xi}^i \quad ; \quad \hat{\xi}^i \rightarrow \xi^i - \hat{\xi}^i. \tag{3.21}$$

This  $\beta$ -symmetry proves useful when studying properties of 3-cycles on  $S^3 \times S^3$  [11].

### 3.1 D6-branes on $S^3 \times S^3$ and Bianchi identity for $F_2$

When  $dF_2 \neq 0$ , the Bianchi identity for the RR 2-form can still be fulfilled by adding appropriate sources. The task is to find the 3-form  $A_3$  that satisfies (2.8) and is the Poincaré dual of the 3-cycles wrapped by the sources.

In general,  $A_3$  is some combination of the 3-forms of the internal space so that it is important to characterize these forms, specially knowing that  $A_3$  must be exact. For  $S^3 \times S^3$  the third Betti number is equal to two and the third cohomology is rather simple. The two representative closed 3-forms are easier to describe in the  $(\xi^i, \hat{\xi}^i)$  basis. In fact, they are basically the volume forms of each  $S^3$ , namely

$$h = \frac{\xi^{123}}{(4ab_1b_2b_3)^{3/4}} \quad ; \quad \hat{h} = -\frac{\hat{\xi}^{123}}{(4ab_1b_2b_3)^{3/4}}. \tag{3.22}$$

The normalization has been chosen so that

$$h \wedge \hat{h} = \frac{J^3}{6t^3} = \eta^{123456}. \tag{3.23}$$

From the six remaining 3-forms that can be constructed there are three exact combinations given by  $d(\xi^i \wedge \hat{\xi}^i)$ . The corresponding forms in terms of the  $\eta^M$  basis are found using the map (3.13). In particular, it follows that

$$a\eta^{456} = b_1\eta^{423} = b_2\eta^{153} = b_3\eta^{126} = \left(\frac{ab_1b_2b_3}{64}\right)^{1/4} (h + \hat{h}), \tag{3.24}$$

where each equality is modulo exact forms.

Let us now study the homology. Our discussion resembles that in [23] and [11]. In  $S^3 \times S^3$  we can identify three special 3-cycles as explained below.

1. The locus  $\hat{\xi}^i = 0$ . By definition this is the first 3-sphere  $S_1^3$ . From the metric (3.16),

$$ds_6^2|_{\hat{\xi}^i=0} = ds_3^2(S_1^3) = \frac{t}{\sqrt{3ab_1b_2b_3}} (\xi^i)^2. \tag{3.25}$$

From the  $\Omega$  form we find that  $\text{Im } \Omega|_{\hat{\xi}^i=0} = 0$ , and moreover

$$\text{Re } \Omega|_{\hat{\xi}^i=0} = -\frac{t^{3/2}}{(3ab_1b_2b_3)^{3/4}} \xi^{123} = -\text{dvol}(S_1^3) . \quad (3.26)$$

This shows that the charge of a brane wrapping  $S_1^3$  is  $-1$ , it would be an anti D6-brane in our conventions. For a D6-brane the 3-sphere must be wrapped in reverse orientation. We will define the corresponding 3-cycle to be  $D_1 = (-S_1^3)$ .

2. The locus  $\xi^i = 0$ . By definition this is the second sphere  $S_2^3$ . We now find that

$$\text{Re } \Omega|_{\xi^i=0} = -\text{dvol}(S_2^3) . \quad (3.27)$$

Thus, a brane wrapping  $S_2^3$  has charge  $-1$  and it is an anti D6-brane in our conventions. Since  $\text{Im } \Omega|_{\xi^i=0} = 0$ , we surmise that the supersymmetric D6-brane must wrap the 3-cycle  $D_2 = (-S_2^3)$ .

3. The locus  $\xi^i = \hat{\xi}^i$ . By definition this is the diagonal 3-sphere  $S_D^3$ . It is easy to check that  $\text{Im } \Omega|_{\xi^i=\hat{\xi}^i} = 0$ . Besides, from the metric (3.16) and the  $\Omega$  form we deduce

$$\text{Re } \Omega|_{\xi^i=\hat{\xi}^i} = \text{dvol}(S_D^3) . \quad (3.28)$$

Due to some extra factors now there is a plus sign in front so that the charge of a brane wrapping the diagonal 3-sphere is a D6-brane with charge  $+1$ . We will denote  $D_0 = S_D^3$ .

The three 3-cycles discussed above,  $D_0$ ,  $D_1$  and  $D_2$ , cannot be independent since the third Betti number of  $S^3 \times S^3$  is two. In fact there is a linear relation among these cycles that will become clear when we discuss the corresponding dual 3-forms.

In general, given a 3-form  $X$  integrated over one of the 3-cycles  $D_i$ , the Poincaré dual form  $Y_i$  to  $D_i$  in  $M_6 = S^3 \times S^3$  is such that

$$\int_{D_i} X = \int_{M_6} X \wedge Y_i . \quad (3.29)$$

For example, for  $D_1 = (-S_1^3)$  we find

$$Y_1 = -\frac{\hat{h}}{\sqrt{V_6}} , \quad (3.30)$$

where  $\hat{h}$  is defined in (3.22). To demonstrate this we can choose

$$X = \text{dvol}(D_1) = -\frac{t^{3/2}}{(3ab_1b_2b_3)^{3/4}} \xi^{123} = -\frac{V_3}{(4\pi)^2} \xi^{123} , \quad (3.31)$$

so that  $\int_{D_1} X = V_3$ . On the other hand we can also compute

$$\int_{M_6} X \wedge \left( -\frac{\hat{h}}{\sqrt{V_6}} \right) = V_3 . \quad (3.32)$$

In a similar fashion we obtain the dual to  $D_2 = (-S_2^3)$  to be

$$Y_2 = -\frac{h}{\sqrt{\mathcal{V}_6}}, \tag{3.33}$$

where  $h$  is defined in (3.22).

We can now compute the intersection number of the 3-cycles  $D_1$  and  $D_2$  by means of the representative dual 3-forms. This is

$$D_2 \cdot D_1 = \int_{D_1} Y_2 = \int_{M_6} Y_2 \wedge Y_1 = \frac{1}{\mathcal{V}_6} \int_{M_6} h \wedge \hat{h} = 1. \tag{3.34}$$

This agrees with the analysis of [23].

We still need to find the dual 3-form of the diagonal 3-sphere  $D_0$ . In this case it is convenient to use the  $\eta^M$  basis. We notice that  $\xi^i = \hat{\xi}^i$  amounts to going to the locus  $\eta^4 = \eta^5 = \eta^6 = 0$ . Either by changing variables or by evaluating directly in (3.9), we obtain

$$\text{dvol}(D_0) = \frac{t^{3/2}}{\sqrt{\tau_1 \tau_2 \tau_3}} \eta^{123}. \tag{3.35}$$

It then follows that the dual 3-form is given by

$$Y_0 = \frac{a(4ab_1b_2b_3)^{1/2}}{4\pi^2} \eta^{456}, \tag{3.36}$$

where we have used that  $\tau_1\tau_2\tau_3 = (27/ab_1b_2b_3)^{1/4}$  as implied by (3.10).

As mentioned before, there must be a linear relation among the three supersymmetric 3-cycles that have been identified. The claim is that

$$D_0 + D_1 + D_2 = 0, \tag{3.37}$$

in homology. This can be simply understood in terms of the dual 3-forms. In fact, from (3.24) we have  $Y_0 = \frac{h+\hat{h}}{\mathcal{V}_6}$ , up to exact forms. Therefore, in cohomology,  $Y_0 + Y_1 + Y_2 = 0$ , modulo exact forms. This confirms the validity of (3.37).

The remaining intersection numbers are also easily calculated. We find for instance  $D_0 \cdot D_2 = \int_{M_6} Y_0 \wedge Y_2 = 1$ . In general,

$$D_i \cdot D_j = \int_{M_6} Y_i \wedge Y_j = \delta_{j,i-1} - \delta_{j,i+1}, \tag{3.38}$$

where the indices are defined modulo 3. These are the intersection numbers found in [23]. In particular they satisfy,  $D_i \cdot (D_0 + D_1 + D_2) = 0$ , consistent with (3.37).

We will now carry the discussion in the quotient space  $S^3 \times S^3/\Gamma$  with  $\Gamma = \mathbb{Z}_2^3$ . To the 3-cycles,  $D_i$  in the covering space we associate  $D'_i$  with corresponding dual forms  $Y'_i$  in the quotient. Closely following [23], let us assume that the lifting to the covering space  $M_6 = S^3 \times S^3$  is given by the map

$$\begin{aligned} (Y'_0, Y'_1, Y'_2) &\rightarrow (Y_0, 8Y_1, 8Y_2) \\ (D'_0, D'_1, D'_2) &\rightarrow (D_0, 8D_1, 8D_2). \end{aligned} \tag{3.39}$$

With this Ansatz we then obtain for instance,

$$\begin{aligned}
 D'_2 \cdot D'_1 &= \int_{\mathcal{S}_6} Y'_2 \wedge Y'_1 = \int_{\mathcal{S}_6} 8Y_2 \wedge 8Y_1 = \frac{1}{8} \int_{M_6} 8Y_2 \wedge 8Y_1 = 8 \\
 D'_0 \cdot D'_2 &= \int_{\mathcal{S}_6} Y'_0 \wedge Y'_2 = \int_{\mathcal{S}_6} Y_0 \wedge 8Y_2 = \frac{1}{8} \int_{M_6} Y_0 \wedge 8Y_2 = 1,
 \end{aligned}
 \tag{3.40}$$

where we have defined  $\mathcal{S}_6 = M_6/\Gamma = S^3 \times S^3/\Gamma$  to streamline expressions. As expected, this is consistent with the normalization

$$\int_{\mathcal{S}_6} \eta^{123456} = \frac{\mathcal{V}_6}{8} = \mathcal{C}
 \tag{3.41}$$

where  $\mathcal{C}t^3$  is the volume of  $\mathcal{S}_6$ . We will see that these intersection numbers also arise in our model 1 in  $D=4$  discussed in appendix A.

According to [23], the 3-cycle  $D'_0$  corresponds to  $D'_0 = S_D^3/\Gamma$ . Namely,  $D_0 = S_D^3$  is an 8-fold cover of  $D'_0$ . Since cycles are not independent, this indicates that wrapping  $N$  D6-branes around each of the cycles  $D'_i$  with  $i = 1, 2$ , requires wrapping  $D'_0$   $8N$  times. In other words,

$$8D'_0 + D'_1 + D'_2 = 0,
 \tag{3.42}$$

which is true by virtue of the map (3.39) and the relation (3.37).

With all the information collected so far we can already establish a connection to our model 1 explained in appendix A. In this model, with mass parameter  $F_0 = 0$ , we found that tadpoles could be cancelled by a setup of supersymmetric D6-branes wrapping particular factorizable 3-cycles in the  $\eta^M$  basis. The concrete configuration is summarized in table 1 where the 3-cycles are explicitly given. It consists of a stack of  $8N_B$  D6-branes wrapping a cycle  $\Pi_A$ ,  $N_B$  D6-branes wrapping a cycle  $\Pi_B$ , plus  $N_B$  D6-branes wrapping the mirror cycle  $\tilde{\Pi}_B$ . In the model, the geometric flux parameters satisfy  $a = b_i = 2N_B/c$ , where  $c$  is related to the RR 2-form background. Interestingly enough, it is possible to represent these factorizable cycles in terms of the supersymmetric 3-cycles in  $S^3 \times S^3/\Gamma$ . In fact, the following identifications can be made

$$\Pi_A = (1, 0)^3 \equiv D'_0 \quad ; \quad \Pi_B = (-1, 1)^3 \equiv D'_2 \quad ; \quad \tilde{\Pi}_B = (-1, -1)^3 \equiv D'_1
 \tag{3.43}$$

Evidence for these matchings comes from the equivalence of the loci described in both the  $\eta^M$  and the  $(\xi^i, \hat{\xi}^i)$  basis, and from agreement of the intersection numbers. For instance,  $\Pi_A \cdot \Pi_B = 1 = D'_0 \cdot D'_2$  and  $\Pi_B \cdot \tilde{\Pi}_B = 8 = D'_2 \cdot D'_1$ . Besides, below we will check that the corresponding dual 3-forms do coincide.

Based on the above results from the analysis of supersymmetric 3-cycles in  $S^3 \times S^3/\Gamma$  we conclude that a setup of D6-branes wrapping the cycles  $D'_0$ ,  $D'_1$  and  $D'_2$ , will lead to tadpole cancellation. Otherwise stated, the corresponding dual 3-forms must add up to the precise 3-form  $A_3$  needed to saturate the Bianchi identity. To substantiate this claim we will examine the Bianchi identity for the RR 2-form in more detail. The starting point is equation (A.20). For sources wrapping space-time the RR 7-form can be written

as  $C_7 = \text{dvol}_4 \wedge X$ , where  $X$  is some 3-form in the internal space which we take to be  $\mathcal{S}_6 = \mathbb{S}^3 \times \mathbb{S}^3/\Gamma$ . Then, (A.20) leads to

$$\int_{\mathcal{S}_6} X \wedge (d\bar{F}_2 - F_0 \bar{H}) + \sqrt{c} \sum_a N_a Q_a \int_{\Pi_a} X = 0 . \quad (3.44)$$

The factor of  $\sqrt{c}$  is necessary because we are writing  $d\bar{F}_2$  and  $\bar{H}$  in a basis of forms with normalization (3.41) or analogous in terms of the  $(\xi^i, \hat{\xi}^i)$  1-forms.

To continue, recall that  $\int_{\Pi_a} X = \int_{\mathcal{S}_6} X \wedge Y'_a$ , where the 3-form  $Y'_a$  is the Poincaré dual of the 3-cycle  $\Pi_a$ . Thus, from the above integral we arrive at the BI

$$d\bar{F}_2 - F_0 \bar{H} + \sqrt{c} \sum_a N_a Q_a Y'_a = 0 . \quad (3.45)$$

In terms of the notation in section 2 we have

$$A_3 = \sum_a N_a Q_a A_3^a , \quad (3.46)$$

where  $A_3^a = \sqrt{c} Y'_a$  is the contribution of each individual source. Recall that  $N_a$  is the number of D6-branes or O6-planes wrapping the 3-cycle  $\Pi_a$  and  $Q_a$  is the corresponding charge.

In the following we focus on the massless case  $F_0 = 0$  as in model 1 of appendix A. As argued in section 2, when  $m = 0$ , necessarily sources of positive charge must be included to satisfy the BI. In this case, in our  $\mathbb{S}^3 \times \mathbb{S}^3/\mathbb{Z}_2^3$  compactification, from previous results we know that  $d\bar{F}_2$  is given by

$$d\bar{F}_2 = -\frac{c}{t} dJ = -\frac{3c}{2t} \mathcal{W}_1 \text{Im} \Omega . \quad (3.47)$$

In the  $\eta^M$  basis this yields the rather simple expansion

$$d\bar{F}_2 = -c(3a\eta^{456} - b_1 \eta^{423} - b_2 \eta^{153} - b_3 \eta^{126}) . \quad (3.48)$$

Our results for tadpole cancellation in model 1 in appendix A suggest a solution to the BI,  $d\bar{F}_2 + A_3 = 0$ . Concretely we propose that in this situation  $A_3$  can be written as

$$A_3 = N_B(8A_3^A + A_3^B + \tilde{A}_3^B) , \quad (3.49)$$

because  $N_A = 8N_B$  and  $Q_A = Q_B = 1$ . Indeed, it is straightforward to check that the BI is satisfied with

$$\begin{aligned} A_3^A &= \eta^{456} , \\ A_3^B &= -(\eta^{456} + \eta^{423} + \eta^{153} + \eta^{126} + \eta^{123} + \eta^{156} + \eta^{426} + \eta^{453}) , \\ \tilde{A}_3^B &= -(\eta^{456} + \eta^{423} + \eta^{153} + \eta^{126} - \eta^{123} - \eta^{156} - \eta^{426} - \eta^{453}) , \end{aligned} \quad (3.50)$$

as long as  $a = b_i = 2N_B/c$ , which precisely guarantees tadpole cancellation.

To close our argument we compare the dual 3-forms  $Y'_a$  with those found before for the supersymmetric 3-cycles in  $S^3 \times S^3/\mathbb{Z}_2^3$ . We find

$$\begin{aligned} Y'_A &= \frac{1}{\sqrt{\mathcal{C}}} A_3^A = \frac{(ab_1b_2b_3)^{3/4}}{2\pi^2} \eta^{456} = Y_0 = Y'_0 \\ Y'_B &= \frac{1}{\sqrt{\mathcal{C}}} A_3^B = 8 \left( -\frac{h}{\sqrt{\mathcal{V}_6}} \right) = 8Y_2 = Y'_2 \\ \tilde{Y}'_B &= \frac{1}{\sqrt{\mathcal{C}}} \tilde{A}_3^B = 8 \left( -\frac{\hat{h}}{\sqrt{\mathcal{V}_6}} \right) = 8Y_1 = Y'_1 . \end{aligned} \tag{3.51}$$

Therefore, the cycles wrapped by D6-branes correspond to the “quotient spheres”  $D'_0$ ,  $D'_1$  and  $D'_2$ , as already anticipated in (3.43).

As we might suspect, a more generic solution to the BI can be obtained as we now explain. Again in the massless case, the problem is to solve

$$d\bar{F}_2 + \sqrt{\mathcal{C}} \sum_a N_a Q_a Y'_a = 0 . \tag{3.52}$$

In general we can attempt a solution with 3 stacks of D6-branes wrapping the supersymmetric quotient 3-spheres so that

$$A_3 = \sqrt{\mathcal{C}} \sum_a N_a Q_a Y'_a = \sqrt{\mathcal{C}} (N_0 Y'_0 + N_1 Y'_1 + N_2 Y'_2) , \tag{3.53}$$

setting the charges to 1. Now, as suggested by (3.42), we choose  $N_0 = 8N$ ,  $N_1 = N_2 = N$ . Then,

$$A_3 = 8\sqrt{\mathcal{C}} N (Y_0 + Y_1 + Y_2) = \frac{2N}{(ab_1b_2b_3)^{1/4}} (3a\eta^{456} - b_1\eta^{423} - b_2\eta^{153} - b_3\eta^{126}) , \tag{3.54}$$

where we used the lifting (3.39) and the expansions of the dual forms in the  $\eta^M$  basis. Comparing with (3.48) we see that the BI is satisfied provided that

$$c = \frac{2N}{(ab_1b_2b_3)^{1/4}} . \tag{3.55}$$

In the  $D=4$  formulation developed in section A.1, this generic solution can be associated to a particular configuration of supersymmetric D6-branes similar to model 1. The setup consists of  $N_B$  D6-branes wrapping  $\Pi_B = (-1, k) \otimes (-1, \ell) \otimes (-1, m)$ , where  $(k, \ell, m)$  are positive integers,  $N_B$  D6-branes along the mirror 3-cycle  $\tilde{\Pi}_B$ , plus  $N_A = 8N_B$  D6-branes wrapping  $\Pi_A = (1, 0)^3$ . It is not difficult to check that tadpoles are cancelled, and  $\Pi_B$  is supersymmetric, as long as  $ac = 2N_B$ ,  $b_1c = 2N_B\ell m$ ,  $b_2c = 2N_Bkm$  and  $b_3c = 2N_Bk\ell$ . Combining these parameters we reproduce (3.55) with  $N = N_B\sqrt{k\ell m}$ .

To finish this section we would like to comment on the massless spectrum originating from the configuration of D6-branes. The interpretation is that in  $S^3 \times S^3/\Gamma$  a setup of  $N_B$  D6-branes wrapping each of the cycles  $D'_1$  and  $D'_2$ , as well as  $N_A = 8N_B$  D6-branes

wrapping  $D'_0$ , allows to satisfy the BI. These D6-branes produce an anomaly-free spectrum with gauge group  $U(N_A) \times U(N_B) \times U(N_B)$  and massless matter content

$$(\mathbf{N}_A, \overline{\mathbf{N}}_B, \mathbf{1}) + (\overline{\mathbf{N}}_A, \mathbf{1}, \mathbf{N}_B) + 8(\mathbf{1}, \mathbf{N}_B, \overline{\mathbf{N}}_B), \quad (3.56)$$

consistent with the intersection numbers of the 3-cycles. Notice that the spectrum is chiral and, therefore it cannot be continuously deformed away. This signals the stability of the D6-brane configuration.

The above spectrum is the same as in model 1 in appendix A. We are assuming that the curvature of the 3-spheres wrapped by the D6-branes is large. In fact, the radius is controlled by the size modulus  $t$  whose vev can turn out large, for instance by adjusting the RR flux  $e_0$  [1]. On the other hand, the fact that the D6-branes wrap 3-spheres can have interesting consequences. For instance, since the first Betti number of  $S^3$  is zero, open string massless scalar moduli are not expected. In the lines of [24] these, adjoint, scalars would become massive through  $\mu$  terms in the effective superpotential<sup>2</sup>. This could be an appealing feature from a phenomenological perspective.

So far we have concentrated here in massless type IIA without orientifold planes. Extensions to more general cases can in principle be worked out and could lead to attractive models from the phenomenological point of view.

#### 4. Flux compactification on $AdS_4 \times CP^3$

Compactification of massless type IIA supergravity on  $AdS_4 \times CP^3$  have been studied in detail in [12–14]. The idea was to look for solutions similar to the Freund-Rubin compactification of eleven-dimensional supergravity. Thus, a non-trivial background for the RR 4-form,  $F_4 \propto \text{dvol}_4$  is turned on. By Hodge duality this is equivalent to  $F_6 \propto \text{dvol}_6$ . The solutions are unwarped and have constant dilaton. There is no  $H$  flux. The RR 2-form flux can be chosen to be  $F_2 \propto J$ , where  $J$  is the fundamental form of  $CP^3$ . When the internal metric is given by the Fubini-Study metric the equations of motion are satisfied. Furthermore the Bianchi identity for  $F_2$  is automatic because  $dJ = 0$ . It can be shown that an extended  $\mathcal{N}=6$  supersymmetry is preserved.

Applying the general results reviewed in section 2 we can see that for  $m = 0$  there is a solution with  $\mathcal{N}=1$  supersymmetry when the metric in  $CP^3$  is chosen to be nearly-Kähler. However, in this case the Bianchi identity for  $F_2 \propto J$  is not satisfied because  $dJ \neq 0$ . Presumably the tadpoles could be cancelled by adding D6-branes. The third homology of  $CP^3$  is trivial but there could exist supersymmetric 3-cycles.

Yet another  $\mathcal{N}=1$  solution with  $m = 0$  can be found by choosing the metric on  $CP^3$  and the RR 2-form flux to descend from the metric of the squashed seven-sphere which gives an  $\mathcal{N}=1$  solution of  $D=11$  supergravity [25]. In this case the  $CP^3$  metric is not Einstein and therefore not nearly-Kähler. According to the general analysis it must be that the metric is such that the two torsion classes  $\mathcal{W}_1$  and  $\mathcal{W}_2$  are different from zero. In fact setting

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<sup>2</sup>We thank P. Cámara for these observations.

$\widehat{\Omega} = -i\Omega$  in (2.3) tells us that

$$dJ = \frac{3}{2}\mathcal{W}_1\text{Im}\Omega \quad ; \quad d\Omega = \mathcal{W}_1J^2 + \mathcal{W}_2 \wedge J, \quad (4.1)$$

where  $\mathcal{W}_1 = \frac{4}{3}\tilde{m}e^{-A}$  and  $\mathcal{W}_2$  is a real primitive 2-form. Substituting in (2.5) then gives

$$F_2 = -\frac{1}{4}\mathcal{W}_1J + *(\mathcal{W}_2 \wedge J), \quad (4.2)$$

where we have put the warp factor to zero. In principle it is then feasible to attain  $dF_2 = 0$  even if  $dJ \neq 0$ . Below we try to check these claims.

The generic metric on  $\mathbb{CP}^3$  can be constructed as a bundle with base  $S^4$  and fiber  $S^2$ . Denoting by  $(\theta, \varphi)$  the coordinates of the  $S^2$  this means that

$$ds_6^2 = d\tilde{s}_4^2 + \lambda^2(d\theta - \sin\varphi\mathcal{A}^1 + \cos\varphi\mathcal{A}^2)^2 + \lambda^2\sin^2\theta\left(d\varphi - \frac{\cos\theta}{\sin\theta}(\cos\varphi\mathcal{A}^1 + \sin\varphi\mathcal{A}^2) + \mathcal{A}^3\right)^2, \quad (4.3)$$

where  $d\tilde{s}_4^2$  is the line element of  $S^4$  and  $\mathcal{A}^A$  is the self-dual  $SU(2)$  instanton potential on  $S^4$ . More explicitly,

$$d\tilde{s}_4^2 = d\psi^2 + \frac{1}{4}\sin^2\psi\Sigma^A\Sigma^A \quad ; \quad \mathcal{A}^A = \cos^2\frac{\psi}{2}\Sigma^A. \quad (4.4)$$

The  $\Sigma^A$ ,  $A = 1, 2, 3$ , are left-invariant  $SU(2)$  1-forms for which we use coordinates

$$\begin{aligned} \Sigma^1 &= \cos\gamma d\alpha + \sin\gamma\sin\alpha d\beta, \\ \Sigma^2 &= -\sin\gamma d\alpha + \cos\gamma\sin\alpha d\beta, \\ \Sigma^3 &= d\gamma + \cos\alpha d\beta, \end{aligned} \quad (4.5)$$

Notice that  $d\Sigma^A = -\frac{1}{2}\epsilon_{ABC}\Sigma^B \wedge \Sigma^C$ .

In the following we will employ a flat Sechsbein defined as

$$\begin{aligned} e^1 &= d\psi \quad ; \quad e^j = \frac{1}{2}\sin\psi\Sigma^{j-1}, \quad j = 2, 3, 4, \\ e^5 &= \lambda(d\theta - \sin\varphi\mathcal{A}^1 + \cos\varphi\mathcal{A}^2), \\ e^6 &= \lambda\sin\theta\left(d\varphi - \frac{\cos\theta}{\sin\theta}(\cos\varphi\mathcal{A}^1 + \sin\varphi\mathcal{A}^2) + \mathcal{A}^3\right). \end{aligned} \quad (4.6)$$

In the flat basis the Ricci tensor of the  $\mathbb{CP}^3$  metric is diagonal with components

$$R_{ab} = (3 - \lambda^2)\delta_{ab} \quad ; \quad R_{ij} = \left(\lambda^2 + \frac{1}{\lambda^2}\right)\delta_{ij}, \quad (4.7)$$

where  $a, b = 1, \dots, 4$ , and  $i, j = 5, 6$ .

Taking  $\lambda^2 = 1$  gives the standard Einstein metric that is Kähler. A second Einstein metric that is nearly-Kähler is obtained setting  $\lambda^2 = \frac{1}{2}$ . In both cases the Einstein equation of motion of type IIA supergravity can be solved with  $F_2 \propto J$ . Another solution can be



found choosing  $\lambda^2 = \frac{1}{5}$  and turning on an appropriate RR 2-form flux. Concretely,  $F_2$  must be

$$F_2 = -\lambda \sin \theta \sin \varphi (e^{12} + e^{34}) - \lambda \sin \theta \cos \varphi (e^{13} + e^{42}) - \lambda \cos \theta (e^{14} + e^{23}) - \frac{1}{\lambda} e^{56} . \quad (4.8)$$

It can be checked that  $dF_2 = 0$  and  $\nabla_m F^{mn} = 0$ . Moreover, we will see that  $F_2$  is of the expected form (4.2), with  $\mathcal{W}_2 \neq 0$ . In appendix B we will check that all equations of motion are satisfied and that  $\mathcal{N}=1$  supersymmetry is preserved.

As already stressed in [12–14], the new  $\mathbb{C}\mathbb{P}^3$  compactification of massless IIA supergravity is directly related to compactification of  $D=11$  supergravity on the squashed seven-sphere [25]. Indeed, the metric on the squashed  $S^7$  can be written as<sup>3</sup>

$$ds_7^2 = (\lambda d\tau - A)^2 + ds_6^2 , \quad (4.9)$$

where  $ds_6^2$  is the above metric on  $\mathbb{C}\mathbb{P}^3$  and the gauge potential  $A$  is such that  $dA$  gives precisely the RR 2-form background displayed in (4.8). When  $\lambda^2 = \frac{1}{5}$  this seven-dimensional metric is Einstein and admits only one Killing spinor.

The fundamental forms  $J$  and  $\Omega$  can be obtained from the Killing spinor in six dimensions. Details are presented in appendix B. The main results are

$$\begin{aligned} J &= -\sin \theta \sin \varphi (e^{12} + e^{34}) - \sin \theta \cos \varphi (e^{13} + e^{42}) - \cos \theta (e^{14} + e^{23}) + e^{56} , \\ \text{Re } \Omega &= \cos \theta \cos \varphi (e^{126} + e^{346}) + \cos \theta \sin \varphi (e^{136} + e^{426}) + \sin \varphi (e^{125} + e^{345}) \\ &\quad - \cos \varphi (e^{135} + e^{425}) - \sin \theta (e^{146} + e^{236}) , \\ \text{Im } \Omega &= -\cos \theta \cos \varphi (e^{125} + e^{345}) - \cos \theta \sin \varphi (e^{135} + e^{425}) + \sin \varphi (e^{126} + e^{346}) \\ &\quad - \cos \varphi (e^{136} + e^{426}) + \sin \theta (e^{145} + e^{235}) . \end{aligned} \quad (4.10)$$

These forms satisfy (2.2).

The torsion classes are found after computing the exterior derivatives that turn out to be exactly of the form (4.1) with

$$\mathcal{W}_1 = \frac{2(1 + \lambda^2)}{3\lambda} \quad ; \quad \mathcal{W}_2 \wedge J = \lambda J^2 - 6\lambda e^{1234} . \quad (4.11)$$

Both  $\mathcal{W}_1$  and  $\mathcal{W}_2$  are real. In fact,  $d\text{Im } \Omega = 0$ . We can check that  $\mathcal{W}_2 \wedge J \wedge J = 0$  so that  $\mathcal{W}_2$  is primitive. It also follows that

$$*(\mathcal{W}_2 \wedge J) = 2\lambda J - 6\lambda e^{56} . \quad (4.12)$$

With all this information it is a simple exercise to verify that the RR 2-form  $F_2$  given in (4.8) can indeed be written as (4.2) when  $\lambda^2 = \frac{1}{5}$ .

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<sup>3</sup>The metric on the squashed  $S^7$  is  $ds_7^2 = d\tilde{s}_4^2 + \lambda^2 (d\sigma^A - \mathcal{A}^A)^2$ , where  $\sigma^A$  is a second set of  $SU(2)$  left-invariant 1-forms. To recover (4.9) just set  $\sigma^1 = \sin \varphi d\theta + \sin \theta \cos \varphi d\tau$ ,  $\sigma^2 = -\cos \varphi d\theta + \sin \theta \sin \varphi d\tau$ ,  $\sigma^3 = -d\varphi + \cos \theta d\tau$ .

## 5. Final remarks

The original motivation behind this paper was to identify the internal space implicit in a class of  $\mathcal{N}=1$  type IIA  $\text{AdS}_4$  vacua obtained using the effective  $D=4$  formalism. As we have explained, this internal space turns out to be  $S^3 \times S^3/\mathbb{Z}_2^3$  with a nearly-Kähler metric. This property, together with the structure of background fluxes, is in complete agreement with the general results derived from supersymmetry conditions and equations of motion in  $D=10$ .

Unlike the Minkowski case,  $\text{AdS}_4$   $\mathcal{N}=1$  type IIA compactifications have the peculiarity that the equations of motion can be solved in the absence of orientifold planes of negative tension. In the  $D=4$  approach this can be simply understood from the tadpole cancellation equations that include fluxes and sources [1]. In  $D=10$ , as reviewed in section 2, this follows from the Bianchi identity for the RR 2-form [4]. In the  $S^3 \times S^3/\mathbb{Z}_2^3$  compactification we have found explicit solutions of the tadpole cancellation conditions and used them to construct configurations of D6-branes that allow to solve the Bianchi identity in  $D=10$ .

A second motivation of our work was to find a concrete example of  $\mathcal{N}=1$  type IIA compactification to  $\text{AdS}_4$  in which the internal space is not nearly-Kähler. This possibility is allowed by the general analysis of flux vacua, it corresponds to the case in which both torsion classes  $\mathcal{W}_1$  and  $\mathcal{W}_2$  are different from zero. Previous attempts to construct examples of this sort failed because the Bianchi identity for the RR 2-form could not be fulfilled [2]. Our contribution has been to observe that some solutions of massless type IIA supergravity discovered long time ago [12–15] do fit within the general framework of  $\text{AdS}_4$  flux vacua while the internal manifold does not have a nearly-Kähler metric. We considered compactification on  $\mathbb{C}\mathbb{P}^3$  and showed that both torsion classes  $\mathcal{W}_1$  and  $\mathcal{W}_2$  are different from zero. Moreover, the background of the RR 2-form has the correct expression in terms of the torsion classes. Another example with both  $\mathcal{W}_1$  and  $\mathcal{W}_2$  non zero, already studied in [10], which has as internal space the coset  $\text{SU}(3)/\text{U}(1)^2$ , can be treated along the same lines as in section 4.

In this note we have exemplified the validity and applicability of the effective  $D=4$  approach to uncover properties of  $D=10$  flux vacua. It is clearly desirable to extend the effective formalism to compactifications with more generic internal manifolds. In the future we hope to join efforts in pursuing further research on this interesting subject.

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## A. Effective approach in $D=4$

This appendix serves several purposes. First we give a concise account of the effective action for  $D=4$ ,  $\mathcal{N}=1$  type IIA toroidal orientifolds [26–28]. We then describe to some extent the specific model that turns out to be related to compactification on  $\text{AdS}_4 \times \text{S}^3 \times \text{S}^3$ . We will also show that the results are in complete agreement with those derived from supersymmetry conditions and equations of motion in  $D=10$ . Finally, we discuss tadpole cancellation equations and provide configurations of supersymmetric D6-branes that solve these equations.

In the  $D=4$  effective formalism the analysis of vacua is based on the superpotential generated by RR, NSNS and geometric fluxes. In type IIA orientifolds the flux induced superpotential can be written as

$$W = \int_{\text{M}_6} e^{J_c} \wedge \overline{F} + \Omega_c \wedge (\overline{H} + dJ_c) . \quad (\text{A.1})$$

The complexified forms defined as

$$J_c = B + iJ \quad ; \quad \Omega_c = C_3 + i\text{Re}(e^{-\phi}\Omega) , \quad (\text{A.2})$$

are expanded in the basis of invariant 2 and 3-forms, with coefficients given by the moduli fields. In the model we are considering these fields are the axiodilaton  $S$ , together with three complex structure  $U_i$  and three Kähler moduli  $T_i$ . The relevant expansions are

$$J_c = iT_i\omega_i \quad ; \quad \Omega_c = iS\alpha_0 - iU_i\alpha_i . \quad (\text{A.3})$$

As we saw in section 3,  $J = t_i\omega_i$ . The NSNS 2-form can also be expanded in terms of the  $\omega_i$  as  $B = -v_i\omega_i$ . The  $v_i$  are the Kähler axions and indeed the Kähler moduli are  $T_i = t_i + iv_i$ . For the axiodilaton and complex structure moduli we can substitute (3.9) to obtain

$$\text{Re } S \equiv s = e^{-\phi} \sqrt{\frac{t_1 t_2 t_3}{\tau_1 \tau_2 \tau_3}} \quad ; \quad \text{Re } U_i \equiv u_i = s\tau_j\tau_k , \quad j \neq k . \quad (\text{A.4})$$

The corresponding axions arise from the RR 3-form given by  $C_3 = -\text{Im } S\alpha_0 + \text{Im } U_i\alpha_i$ .

To compute the superpotential we need expansions for the background fluxes. We follow the approach of [1] where the fluxes are chosen to comply with the  $\mathbb{Z}_2 \times \mathbb{Z}_2$  symmetry (3.2). Thus, just as  $J_c$  and  $\Omega_c$ , the fluxes are to be expanded in the basis of invariant forms displayed in (3.5). Furthermore, since we are assuming that the moduli are those of toroidal IIA orientifolds, the fluxes are required to conform to the orientifold involution (3.3). This means that  $\overline{F}_0$  and  $\overline{F}_4$  are even, whereas  $\overline{H}$ ,  $\overline{F}_2$  and  $\overline{F}_6$  are odd under the orientifold involution [26]. The upshot is that background fluxes have the following expansions

$$\begin{aligned} \overline{H} &= h_0\beta_0 + h_i\beta_i \quad ; \quad \overline{F}_0 = -M \quad ; \quad \overline{F}_2 = q_i\omega_i \\ \overline{F}_4 &= e_i\tilde{\omega}_i \quad ; \quad \overline{F}_6 = e_0\alpha_0 \wedge \beta_0 . \end{aligned} \quad (\text{A.5})$$

The exterior derivative of these fluxes and the Kähler form  $J$  are found using (3.4) that define the internal space.

The scalar potential of the moduli has the standard  $\mathcal{N}=1$  expression

$$V = e^K \left\{ \sum_{\Phi=S,T_i,U_i} (\Phi + \Phi^*)^2 |D_\Phi W|^2 - 3|W|^2 \right\}, \quad (\text{A.6})$$

where we already assumed the classical Kähler potential  $K = -\sum_{\Phi=S,T_i,U_i} \log(\Phi + \Phi^*)$ , and  $D_\Phi W = \partial_\Phi W + W \partial_\Phi K$ . Supersymmetric AdS minima are determined by the condition  $D_\Phi W = 0$ .

To obtain the model analyzed in [1] one chooses RR fluxes  $q_i = -c$  and  $e_i = e$  so that a configuration with  $T_i = T$  is allowed. The resulting superpotential is simply<sup>4</sup>

$$\frac{W}{\mathcal{C}} = e_0 + 3ieT + 3cT^2 + iMT^3 + (ih_0 - 3aT)S - \sum_{k=0}^3 (ih_k + b_k T)U_k. \quad (\text{A.7})$$

This superpotential admits supersymmetric AdS minima provided that the fluxes satisfy the constraint

$$3h_k a + h_0 b_k = 0 \quad ; \quad k = 1, 2, 3. \quad (\text{A.8})$$

In this case the real parts of the axiodilaton and complex structure moduli are completely determined in terms of the Kähler modulus according to

$$as = 2t(c - Mv) \quad ; \quad 3as = b_k u_k \quad ; \quad k = 1, 2, 3. \quad (\text{A.9})$$

Recall that  $s = \text{Re } S$ ,  $u_k = \text{Re } U_k$ ,  $t = \text{Re } T$  and  $v = \text{Im } T$  and that the real part of the moduli are positive definite. Thus, (A.9) requires that the geometric fluxes  $a$  and  $b_k$  be of the same sign. For the  $S$  and  $U_i$  axions only one linear combination is fixed, this is

$$3a \text{Im } S + b_k \text{Im } U_k = 3e + \frac{3c}{a}(3h_0 - 7av) - \frac{3M}{a}v(3h_0 - 8av). \quad (\text{A.10})$$

To have the minimum with  $T_i = T$  it must also be that  $b_1 \text{Im } U_1 = b_2 \text{Im } U_2 = b_3 \text{Im } U_3$ .

The vacuum expectation value for the Kähler modulus depends on whether the mass parameter  $M$  vanishes or not. When  $M = 0$  it is found that

$$v = v_0 = \frac{h_0}{3a} \quad ; \quad 9ct^2 = e_0 - \frac{h_0 e}{a} - \frac{h_0^2 c}{3a^2}. \quad (\text{A.11})$$

In this case (A.9) implies that necessarily there is a flux of the RR 2-form, i.e.  $c \neq 0$ , and furthermore that  $ac > 0$  and  $cb_k > 0$ . Background fluxes  $\overline{H}$  and  $\overline{F}_4$  can be absent but then the Freund-Rubin flux  $\overline{F}_6 \sim e_0$  must be turned on.

When  $M \neq 0$  the Kähler axion satisfies the cubic equation

$$160(v-v_0)^3 + 294 \left( v_0 - \frac{c}{M} \right) (v-v_0)^2 + 135 \left( v_0 - \frac{c}{M} \right)^2 (v-v_0) + v_0^2 \left( v_0 - \frac{3c}{M} \right) + \frac{1}{Ma} (e_0 a - e h_0) = 0. \quad (\text{A.12})$$

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<sup>4</sup>A volume factor  $\mathcal{C}$  appears here due to normalization (3.41).

The real part of the Kähler modulus is now determined from

$$t^2 = 15\left(v - \frac{c}{M}\right)(v - v_0) . \tag{A.13}$$

The solution for  $v$  must be real and such that  $t^2 > 0$ .

Let us now check that the above results agree with the general analysis in  $D=10$ . To begin observe that we have  $d\text{Im}\Omega = 0$  compared to  $d\text{Re}\widehat{\Omega} = 0$ . We find that in order to match the  $D=4$  and  $D=10$  results we need to make the choice

$$\widehat{\Omega} = -i\Omega \quad ; \quad \alpha + \beta = 0 \text{ mod } 2\pi . \tag{A.14}$$

The full exterior derivatives of  $J$  and  $\Omega$  are given in (3.11).

The next step is to express the field strengths in terms of the background fluxes and the moduli. In the case at hand, with  $q_i = -c$ ,  $e_i = e$ ,  $T_i = T$ , it is possible to write most forms in terms of  $J$  and  $\Omega$ . For example,  $B = -vJ/t$ ,  $\overline{F}_4 = eJ^2/2t^2$ , and so on. After substituting in (2.6) we find

$$\begin{aligned} H &= \frac{se^\phi}{t^3}(h_0 - 3av)\text{Im}\Omega , \\ F_2 &= \frac{Mv - c}{t} J , \\ F_4 &= [3e + 6cv - 3Mv^2 - (3a\text{Im}S + b_i\text{Im}U_i)] \frac{J^2}{6t^2} , \\ F_6 &= \left[ e_0 - 3ev - 3cv^2 + Mv^3 + \left(v - \frac{h_0}{3a}\right)(3a\text{Im}S + b_i\text{Im}U_i) \right] \frac{J^3}{6t^3} . \end{aligned} \tag{A.15}$$

All these expressions greatly simplify upon using (A.10) and (A.12). In the end we obtain

$$\begin{aligned} dJ &= \frac{6(c - Mv)}{t} e^\phi \text{Im}\Omega \quad ; \quad d\Omega = \frac{4(c - Mv)}{t} e^\phi J^2 \quad ; \quad H = \frac{2}{5} M e^\phi \text{Im}\Omega , \\ F_0 &= -M \quad ; \quad F_2 = \frac{Mv - c}{t} J \quad ; \quad F_4 = -\frac{3M}{10} J^2 \quad ; \quad F_6 = \frac{3(c - Mv)}{2t} J^3 . \end{aligned} \tag{A.16}$$

These agree with (2.3) and (2.5) provided that

$$\phi = 3A \quad ; \quad m = \frac{M}{5} e^{4A} \quad ; \quad \tilde{m} = \frac{3(c - Mv)}{t} e^{4A} . \tag{A.17}$$

The relation between the dilaton and the warp factor is precisely the same found in the ten-dimensional analysis.

It is also interesting to compute the cosmological constant which follows from the value  $V_0$  of the scalar potential at the minimum. For the AdS minimum,  $V_0 = -3e^K|W_0|^2$ . To determine  $W_0$  we can substitute the vevs of the moduli in (A.7). A more general approach is to use the original form of the superpotential (A.1). Using previous results to evaluate the integrand at the minimum we arrive at

$$e^{Jc} \wedge \overline{F} + \Omega_c \wedge (\overline{H} + dJ_c) \Big|_0 = \frac{2i}{3} (m + i\tilde{m}) e^{-4A} J^3 . \tag{A.18}$$

This shows that the superpotential at the minimum is proportional to the complex constant  $\mu$ . More precisely,  $|W_0|^2 = 16t^6 e^{-8A} |\mu|^2 \mathcal{C}^2$ . For the classical Kähler potential,  $e^K = (2^7 t^3 s u_1 u_2 u_3 \mathcal{C})^{-1}$ , which can be rewritten as  $e^K = e^{4\phi} / 128 t^9 \mathcal{C}^3$ . Therefore,

$$V_0 = -\frac{3e^{4A}}{8\mathcal{C}t^3} |\mu|^2 = \frac{\Lambda}{M_P^2}. \quad (\text{A.19})$$

where  $\Lambda = -3|\mu|^2$  is the cosmological constant and  $M_P^2 = 8e^{2A-2\phi} \mathcal{C}t^3$ . The moduli above are evaluated at the minimum and we are taking  $\alpha' = 1$ .

### A.1 Tadpole cancellation and D6-branes

The fluxes induce tadpoles for the RR 7-form  $C_7$  that can also couple to D6-branes and O6-planes. In general these objects span space-time and wrap a 3-cycle in  $M_6$ . The RR 7-form can then be written as  $C_7 = \text{dvol}_4 \wedge X$ , where  $X$  is some 3-form in the internal space, which can be expanded in a convenient basis. We denote by  $\Pi_a$  the 3-cycle wrapped by a stack of  $N_a$  D6<sub>a</sub>-branes or O6<sub>a</sub>-planes. The coupling of  $C_7$  in the action has contributions from fluxes and from the sources, namely

$$\int_{M_4 \times \mathcal{S}_6} C_7 \wedge (d\bar{F}_2 - F_0 \bar{H}) + \sqrt{\mathcal{C}} \sum_a N_a Q_a \int_{M_4 \times \Pi_a} C_7, \quad (\text{A.20})$$

where  $Q_a = 1$  for D6-branes and  $Q_a = -4$  for O6-planes. Here we are considering the internal space to be  $\mathcal{S}_6 = S^3 \times S^3 / \mathbb{Z}_2^3$ . The factor  $\sqrt{\mathcal{C}}$  must be included for consistency with the normalization of the 1-form basis (see 3.41).

As usual in the  $D=4$  effective formulation, it appears useful to consider factorizable 3-chains

$$\Pi_a = (n_a^1, m_a^1) \otimes (n_a^2, m_a^2) \otimes (n_a^3, m_a^3), \quad (\text{A.21})$$

where  $n_a^i$  ( $m_a^i$ ) are the wrapping numbers along the  $\eta^i$  ( $\eta^{i+3}$ ). In particular, there is a basis of 3-chains  $\Pi_{ijk}$  spanning the  $\{i, j, k\}$  directions. For instance,  $\Pi_{123} = (1, 0) \otimes (1, 0) \otimes (1, 0)$ , etc. . . To each  $\Pi_{ijk}$  we have a corresponding ‘‘dual’’ 3-form  $\eta^{ijk}$  such that

$$\int_{\Pi_{i'j'k'}} \eta^{ijk} = \frac{1}{\sqrt{\mathcal{C}}} \delta_{i,i'} \delta_{j,j'} \delta_{k,k'}. \quad (\text{A.22})$$

Integrating over the 3-chain  $\Pi_a$  then gives,  $\int_{\Pi_a} \eta^{123} = \frac{1}{\sqrt{\mathcal{C}}} n_a^1 n_a^2 n_a^3$ ,  $\int_{\Pi_a} \eta^{156} = \frac{1}{\sqrt{\mathcal{C}}} n_a^1 m_a^2 m_a^3$ , and so on.

It is worth noticing that the basis manifolds  $\Pi_{ijk}$  are not necessarily closed cycles and, therefore, neither is  $\Pi_a$ , for generic wrappings. As an example, consider the exact form  $d(\xi^1 \wedge \hat{\xi}^1) = 2\sqrt{ab_1 b_2 b_3} (a\eta^{456} + b_1\eta^{423} - b_2\eta^{153} - b_3\eta^{126})$ , then,  $\int_{\Pi_{456}} d(\xi^1 \wedge \hat{\xi}^1) = \frac{2}{\sqrt{\mathcal{C}}} \sqrt{ab_1 b_2 b_3}$ , indicating that the manifold  $\Pi_{456}$  has a boundary (see [29] for a related discussion). We rely on tadpole cancellation and supersymmetry to restrict to the adequate wrappings for D6-branes. When the orientifold action (3.3) is implemented there are eight O6-planes along  $\otimes_i (1, 0)$  and image D6-branes wrapping  $\otimes_i (n_a^i, -m_a^i)$  must be included.

To preserve the same supersymmetries as the background the D6-branes must wrap cycles  $\Pi_a$  such that

$$\theta_a^1 + \theta_a^2 + \theta_a^3 = 0 \pmod{2\pi}, \quad (\text{A.23})$$

where the angles are measured in accordance with

$$\tan \theta_a^j = \frac{m_a^j \tau_j}{n_a^j} . \tag{A.24}$$

Recall that the  $\tau_i$  are the complex structure moduli that enter in the metric as shown in (3.7). In the vacuum we are considering they satisfy (3.10).

From the supersymmetric constraint (A.23) it follows that

$$\tau_1 \tau_2 \tau_3 m_a^1 m_a^2 m_a^3 - \tau_1 m_a^1 n_a^2 n_a^3 - \tau_2 n_a^1 m_a^2 n_a^3 - \tau_3 n_a^1 n_a^2 m_a^3 = 0 . \tag{A.25}$$

This condition amounts to  $\text{Im} \Omega|_{\Pi_a} = 0$ . In fact, the supersymmetric cycles are calibrated by  $\text{Re} \Omega$  and the condition on the angles is equivalent to  $\text{Re} \Omega|_{\Pi_a} = \text{dvol}(\Pi_a)$ . Besides, the factorizable cycles satisfy  $J|_{\Pi_a} = 0$ .

Substituting the fluxes and the data for the sources in (A.20) we obtain the tadpole cancellation equations. The conditions receiving flux contributions are

$$\begin{aligned} \sum_a N_a Q_a n_a^1 n_a^2 n_a^3 + (M h_0 - 3ac) &= 0 , \\ \sum_a N_a Q_a n_a^1 m_a^2 m_a^3 + (M h_1 + b_1 c) &= 0 , \\ \sum_a N_a Q_a m_a^1 n_a^2 m_a^3 + (M h_2 + b_2 c) &= 0 , \\ \sum_a N_a Q_a m_a^1 m_a^2 n_a^3 + (M h_3 + b_3 c) &= 0 . \end{aligned} \tag{A.26}$$

The sum in  $a$  includes  $O6_a$ -planes, when orientifold actions are performed, as well as  $D6_a$ -branes and their orientifold images if necessary. Finally, there are fluxless conditions

$$\begin{aligned} \sum_a N_a Q_a m_a^1 m_a^2 m_a^3 &= 0 , \\ \sum_a N_a Q_a m_a^1 n_a^2 n_a^3 &= 0 , \\ \sum_a N_a Q_a n_a^1 m_a^2 n_a^3 &= 0 , \\ \sum_a N_a Q_a n_a^1 n_a^2 m_a^3 &= 0 . \end{aligned} \tag{A.27}$$

When the orientifold action (3.3) is implemented these last four constraints are automatically satisfied once images are included.

When  $M \neq 0$  the tadpole equations admit a solution without branes or O-planes. This happens because  $h_k = -h_0 b_k / 3a$  and then all flux tadpoles can cancel simultaneously when  $M h_0 = 3ac$  [1]. Now, the relations (A.17) and (A.13) imply that this condition is equivalent to  $\tilde{m}^2 = 15m^2$ . As explained in section 2 this is precisely the case when the internal space is nearly-Kähler and no sources are necessary to satisfy the Bianchi identity

$N_a$	$(n_a^1, m_a^1)$	$(n_a^2, m_a^2)$	$(n_a^3, m_a^3)$
$N_A$	(1, 0)	(1, 0)	(1, 0)
$N_B$	(-1, 1)	(-1, 1)	(-1, 1)
$N_B$	(-1, -1)	(-1, -1)	(-1, -1)

**Table 1:** Wrapping numbers for D6-branes in model 1

$N_a$	$(n_a^1, m_a^1)$	$(n_a^2, m_a^2)$	$(n_a^3, m_a^3)$
$N_0$	(1, 0)	(1, 0)	(1, 0)
$N_1$	(1, 0)	(0, -1)	(0, 1)
$N_2$	(0, 1)	(1, 0)	(0, -1)
$N_3$	(0, 1)	(0, -1)	(1, 0)

**Table 2:** Wrapping numbers for D6-branes in model 2

for  $F_2$ . In  $D=10$  we have further seen that when  $\tilde{m}^2 > 15m^2$  the Bianchi identity can be satisfied by adding sources of positive charge. In the  $D=4$  approach it is indeed evident that whenever  $Mh_0 < 3ac$  the tadpoles can be cancelled by adding only D6-branes.

To present examples of tadpole cancellation with only D6-branes we will consider the case  $M = 0$  in the  $S^3 \times S^3$  compactification that we have been analyzing. A first model consists of the factorizable D6-branes shown in table 1. The third stack is the mirror image,  $\tilde{m}_B^i = -m_B^i$ , of the second and it is included to saturate the fluxless tadpole equations. We also take  $N_A = 8N_B$ . The first stack has  $\theta_A^i = 0$ , hence it is supersymmetric independently of the values of the complex structure parameters. On the other hand, substituting the wrapping numbers in (A.26) gives the relations  $2N_B = ac = b_1c = b_2c = b_3c$ , needed for tadpole cancellation. Next, using that  $\tau_i = b_i \sqrt{3a/b_1 b_2 b_3}$ , we find  $\tau_1 = \tau_2 = \tau_3 = \sqrt{3}$ . We can then check that the second and third stack are also supersymmetric. In fact, computing  $\tan \theta_B^i$  for the second shows that  $\theta_B^i = 2\pi/3$ . Assuming that the  $\Pi_a$  cycles have large volume, in this model 1 the resulting gauge group is  $U(N_A) \times U(N_B) \times U(N_B)$ . According to the intersections between cycles, the matter content consists of chiral multiplets transforming as

$$(N_A, \overline{N}_B, \mathbf{1}) + (\overline{N}_A, \mathbf{1}, N_B) + 8(\mathbf{1}, N_B, \overline{N}_B) . \tag{A.28}$$

The multiplicity 8 of the last representation arises from the intersection number between the cycle B and its mirror. Since  $N_A = 8N_B$  this chiral spectrum is free of gauge anomalies.

There are other D6-brane configurations capable of canceling tadpoles. Some are equivalent to the setup in model 1 but others belong to a different class. For instance, in table 2 we display a model 2 with four stacks of branes that are all supersymmetric independently of the complex structure moduli. To cancel tadpoles the numbers of branes in each stack must be related to the fluxes by  $N_0 = 3ac$  and  $N_i = b_i c$ . In this model the resulting spectrum is non-chiral.



## B. Supersymmetric vacua of massless type IIA supergravity in $D=10$

In this appendix we tersely summarize some basic aspects of compactification of massless IIA supergravity on  $\text{AdS}_4 \times M_6$ . We will review the case when the internal space is  $\mathbb{CP}^3$  and appropriate fluxes are turned on so that there is a vacuum with  $\mathcal{N}=1$  supersymmetry in  $D=4$  [12–14]. Our main goal is to explicitly find the six dimensional Killing spinor in order to determine the fundamental forms  $J$  and  $\Omega$  that define the  $SU(3)$  structure. We will follow and refer to the discussion of [12] where the equations of motion and the supersymmetry transformations are spelled out in full detail.

We consider a class of vacua with background metric of the product form (2.1) but to simplify the warp factor  $A$  is fixed to zero. The dilaton  $\phi$  is assumed to be constant whereas the NS 2-form and its field strength are taken to vanish. On the contrary, there are non-trivial RR fluxes. For the 4-form one makes the Freund-Rubin Ansatz

$$F_{\mu\nu\alpha\beta} = 3f\epsilon_{\mu\nu\alpha\beta} \quad ; \quad \epsilon_{0123} = \sqrt{-g_4}, \quad (\text{B.1})$$

while other components are zero. For the RR 2-form there is a flux  $F_{mn}$  through  $M_6$  to be specified shortly. Under these conditions the equations of motion reduce to

$$\begin{aligned} R_{\mu\nu} &= -12e^{\phi/2}f^2g_{\mu\nu}, \\ R_{mn} &= 6e^{\phi/2}f^2g_{mn} + \frac{1}{2}e^{3\phi/2}F_{mp}F_n{}^p, \\ e^\phi F_{mn}F^{mn} &= 24f^2 \quad ; \quad \nabla_m F^{mn} = 0. \end{aligned} \quad (\text{B.2})$$

The Einstein equation in  $D=4$  shows that space-time can indeed be taken to be  $\text{AdS}_4$  with cosmological constant  $\Lambda = -12e^{\phi/2}f^2$ .

To characterize the internal space we still need to specify the flux  $F_2$ . We will see that it is consistent to take  $M_6$  to be  $\mathbb{CP}^3$  with metric given in (4.3), while  $F_2$  can be set equal to the 2-form (4.8). This RR 2-form satisfies the equation of motion and the properties

$$F_{mn}F^{mn} = 2\left(2\lambda^2 + \frac{1}{\lambda^2}\right) \quad ; \quad F_{ac}F_b{}^c = \lambda^2\delta_{ab} \quad ; \quad F_{ik}F_j{}^k = \frac{1}{\lambda^2}\delta_{ij}, \quad (\text{B.3})$$

where  $a, b, c = 1, \dots, 4$ , and  $i, j, k = 5, 6$ , are flat indices.

Once the flux  $F_2$  is given, the dilaton equation of motion implies that the vevs  $e^\phi$ ,  $f$  and the metric parameter  $\lambda$  are related by

$$e^\phi\left(2\lambda^2 + \frac{1}{\lambda^2}\right) = 12f^2. \quad (\text{B.4})$$

Substituting in the  $D=6$  Einstein equation we then find that in the flat basis the Ricci tensor is diagonal with components

$$R_{ab} = \frac{1}{2}e^{3\phi/2}\left(3\lambda^2 + \frac{1}{\lambda^2}\right)\delta_{ab} \quad ; \quad R_{ij} = e^{3\phi/2}\left(\lambda^2 + \frac{1}{\lambda^2}\right)\delta_{ij}. \quad (\text{B.5})$$

The Ricci tensor of the generic  $\mathbb{CP}^3$  metric has precisely this structure. Comparing with (4.7) we see that the dilaton vev has to be  $e^\phi = 1$ . Moreover, the parameter  $\lambda$  must be such that

$$5\lambda^2 - 6 + \frac{1}{\lambda^2} = 0. \tag{B.6}$$

There is a solution with  $\lambda^2 = 1$  for which the metric is Einstein. We are more interested in the solution with  $\lambda^2 = 1/5$ . In this case, from (B.4) it transpires that the Freund-Rubin parameter is fixed to be  $f^2 = 9/20$ .

We now discuss the requirements for residual supersymmetry in  $D=4$ . We will employ exactly the same conventions of [12] for the  $D=10$  Dirac matrices. In  $D=6$  we basically adopt the matrices used in [25] in  $D=7$ . With flat indices these are

$$\begin{aligned} \Gamma_1 &= i\gamma_0 \otimes \mathbb{1} \quad ; \quad \Gamma_2 = \gamma_1 \otimes \mathbb{1} \quad ; \quad \Gamma_3 = \gamma_2 \otimes \mathbb{1} \quad ; \quad \Gamma_4 = \gamma_3 \otimes \mathbb{1} \\ \Gamma_5 &= i\gamma_5 \otimes \sigma_1 \quad ; \quad \Gamma_6 = i\gamma_5 \otimes \sigma_2 \quad ; \quad \Gamma_0 = \Gamma_1 \cdots \Gamma_6 = i\gamma_5 \otimes \sigma_3, \end{aligned} \tag{B.7}$$

where  $\sigma_i$  are the Pauli matrices. The 4-dimensional matrices are

$$\gamma_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad ; \quad \gamma_a = \begin{pmatrix} 0 & \sigma_a \\ -\sigma_a & 0 \end{pmatrix}, \tag{B.8}$$

and  $\gamma_5 = -i\gamma_0\gamma_1\gamma_2\gamma_3$ . We will also need the charge conjugation matrix in  $D=6$  which in our conventions is given by  $C = \Gamma_2\Gamma_4\Gamma_6$ .

To study the conditions for the vacuum to preserve supersymmetry we first write the 10-dimensional parameter as  $\epsilon \otimes \eta$ , where  $\epsilon$  and  $\eta$  are respectively spinors in four and six dimensions. We then substitute the vevs of all fields in the supersymmetry transformations of the fermionic fields which in  $D=10$ , IIA supergravity are the gravitino and the dilatino. We refer the reader to reference [12] for the explicit equations of these transformations. From the dilatino variation we obtain

$$(S + 2f)\eta = 0, \tag{B.9}$$

where the matrix  $S$  depends on the RR 2-form flux as

$$S = \frac{1}{2} F_{mn} \Gamma^{mn} \Gamma_0. \tag{B.10}$$

For the  $F_2$  background in (4.8),  $S$  turns out to have eigenvalues  $1/\lambda$ ,  $(2\lambda^2 - 1)/\lambda$ , and  $-(2\lambda^2 + 1)/\lambda$ , with degeneracies 4, 2 and 2 respectively. Remarkably, for the case of interest with  $\lambda^2 = 1/5$  and  $f^2 = 9\lambda^2/4$ ,  $S$  can have an eigenvalue  $-2f$  as long as we take  $f = 3\lambda/2$ . The corresponding eigenvector has the simple form

$$\eta = \begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad ; \quad s_1 = -\frac{\sin \theta e^{-i\phi} s_3}{1 + \cos \theta} \quad ; \quad s_4 = \frac{\sin \theta e^{i\phi} s_2}{1 + \cos \theta}, \tag{B.11}$$

where  $s_2$  and  $s_3$  in principle depend on all internal coordinates.

From the gravitino variation  $\delta\psi_\mu$ , using (B.9), we find

$$D_\mu\epsilon - e^{\phi/4}f\gamma_5\gamma_\mu\epsilon = 0 . \tag{B.12}$$

This is the expected equation for the supersymmetry parameter in  $\text{AdS}_4$  with cosmological constant  $\Lambda = -12e^{\phi/2}f^2$ . Finally, from the variation  $\delta\psi_m$  we obtain the Killing equation

$$D_m\eta - \frac{f}{2}\Gamma_m\eta - \frac{1}{4}F_m{}^n\Gamma_n\Gamma_0\eta = 0 , \tag{B.13}$$

where we have set  $e^\phi = 1$ . For the covariant derivative acting on spinors we use the conventions of [25].

It remains to solve the Killing equation to determine the unknown functions  $s_2$  and  $s_3$  in  $\eta$ . From the  $\psi$  component we find

$$s_2 = ie^{-i\phi} s_3 . \tag{B.14}$$

It further follows that  $s_3$  is completely independent of the  $S^4$  coordinates  $(\psi, \alpha, \beta, \gamma)$ , but depends on the  $S^2$  variables as

$$s_3 = e^{i\delta} e^{-i\phi/2} \cos \frac{\theta}{2} , \tag{B.15}$$

where  $\delta$  is a constant phase. The normalization guarantees that the Weyl spinors

$$\eta_\pm = \frac{1 \pm i\Gamma_0}{2} \eta \tag{B.16}$$

satisfy  $\eta_\pm^\dagger \eta_\pm = 1$ . The phase  $\delta$  is fixed by imposing the reality condition  $\eta_+^* = C \eta_-$ .

We are now ready to compute the fundamental forms  $J$  and  $\Omega$  defined by

$$J_{mn} = i\eta_-^\dagger \Gamma_{mn} \eta_- \quad ; \quad \Omega_{mnp} = \eta_+^\dagger \Gamma_{mnp} \eta_- . \tag{B.17}$$

In the end we obtain the results reported in section 4. We stress that there is a unique Killing spinor  $\eta$  so that the internal manifold has  $\text{SU}(3)$  structure and there is  $\mathcal{N}=1$  supersymmetry in  $D=4$ .

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